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ON AN EXTENSION OF THE MULTIPHASE BMO ALGORITHM TO ANISOTROPIC ENERGIES

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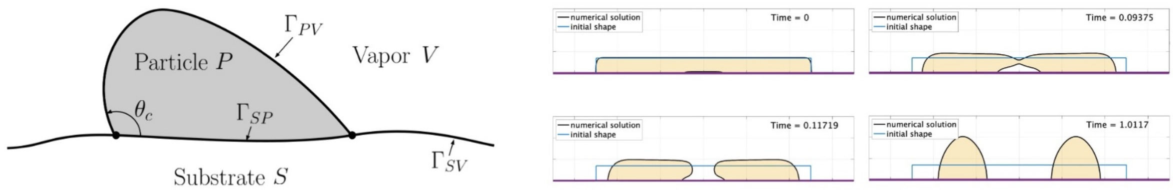
Thresholding schemes, often referred to as the Bence-Merriman-Osher (BMO) algorithm [2] provide a remarkably simple yet powerful way to approximate curvature-driven interface motion. Each time step consists only of a convolution with a kernel followed by a thresholding operation. Despite this apparent simplicity, such schemes admit a deep variational interpretation: they can be viewed as minimizing movements for nonlocal energies that Γ-converge to interfacial energies as the time step tends to zero [5]. This connection has made thresholding methods an attractive tool both for analysis and for numerical simulation of geometric evolution problems involving topology changes.

In this talk, we first revisit the BMO algorithm in its classical form for isotropic mean curvature flow and explain its interpretation as a time discretization of the L^2-gradient flow of the perimeter functional. We then survey several extensions of this idea to multiphase flows, weighted surface tensions, and problems with volume constraints [6, 7], emphasizing how convolution-based schemes can be systematically adapted to increasingly complex interfacial energies. As an illustration of its versatility, we discuss applications to biological modeling, including our recent work on a numerical algorithm for modeling cellular rearrangements in tissue morphogenesis, where thresholding methods allow one to simulate large interfacial networks undergoing frequent topological transitions [9].

The main focus of the talk is a new analytical result concerning anisotropic mean curvature flow of a particle constrained by a rigid, inhomogeneous substrate [3]. The system consists of three interfaces: particle-vapor, particle-substrate, and vapor-substrate, each endowed with its own surface tension, possibly depending on position and, for the free interface, also on orientation. The evolution is given by the L^2-gradient flow of the total anisotropic interfacial energy

$$E(P) = \int_{\Gamma_{PV}} \psi_{PV}(x,\nu)\, d\mathcal{H}^{d-1} + \int_{\Gamma_{SP}} \gamma_{SP}(x)\, d\mathcal{H}^{d-1} + \int_{\Gamma_{SV}} \gamma_{SV}(x)\, d\mathcal{H}^{d-1},$$

leading to a weighted anisotropic mean curvature motion coupled with a generalized Herring contact angle condition at the substrate. The derivation is naturally formulated in the framework of anisotropic (Finsler) geometry following Bellettini-Paolini [1].



Setting of the problem (left) and an example of numerical simulation (right).

Such problems arise naturally in cell motility, coating processes, and solid-state dewetting, where topology changes and heterogeneous surface energies play a crucial role. While related isotropic problems with obstacles have been studied [8, 10], and anisotropic flows without obstacles are well understood in the thresholding framework [4], the combination of anisotropy, spatial inhomogeneity, and an obstacle presents significant analytical challenges. In particular, standard convergence proofs for thresholding schemes rely on monotonicity properties of the approximate energies that fail when different interfaces carry different anisotropies.

We show how this difficulty can be overcome in the present setting. We construct a family of nonlocal approximate energies of the form

$$E_h(u) = \frac{1}{\sqrt{h}} \int \left(\widetilde{\gamma}_{PV} u K_h * (\mathbb{1}_\Omega - u) + \widetilde{\gamma}_{SP} u K_h * \mathbb{1}_S + \widetilde{\gamma}_{SV} \left(\mathbb{1}_\Omega - u\right) K_h * \mathbb{1}_S\right) dx,$$

where a single anisotropic kernel K_h encodes the orientation dependence and suitably modified surface tensions $\widetilde{\gamma}_{ij}$ incorporate the spatial inhomogeneity.

Our main result proves that these energies Γ-converge in L^1 to the desired anisotropic interfacial energy under a strengthened triangle inequality for the surface tensions. This provides the first rigorous step toward an existence theory for anisotropic mean curvature flow with obstacle based on the thresholding approach and suggests a pathway to treat more general multiphase problems with differing anisotropies. We conclude the talk by briefly outlining the proof of convergence of the corresponding thresholding scheme, leading to existence of weak BV solutions to the anisotropic obstacle problem.

Keywords: thresholding scheme, anisotropic mean curvature flow, obstacle problem, Γ-convergence, interfacial energies with spatial inhomogeneity, topology changes in interface evolution

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