

## Finite Element Simulation of Aqueous Humour Flow with Thermal Convection in the Human Eye

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Aqueous humour is a transparent fluid filling the space between the cornea and lens. It is said to be driven not only by production and drainage but also by thermal convection when opening the eye. It plays a crucial role in biological functions and is strongly related to glaucoma, the leading cause of blindness in Japan. However, its flow remains unclear due to its transparency. This study presents a 3D FEM simulation of the flow and estimation using clinical video.

We simulated the 3D aqueous humour flow on a patient-specific tetrahedral mesh derived from clinical images. Over a time interval, the velocity, pressure, and temperature were assumed to be governed by the incompressible Navier–Stokes and heat advection-diffusion equations under the Boussinesq approximation. We accounted for posture-dependent gravity directions and corneal cooling during eye opening. To prevent unintended backflow at the outlet, we applied a Signorini-type backflow prevention term [2]. The governing equations were discretized by the finite element method using P1/P1 elements, integrating pressure stabilization and the method of characteristics [1].

For the validation, we estimated the flow field from clinical videos using an optical flow method. After surgery, red blood cells mixed into the aqueous humour and they acted as tracers. Figure 1 shows simulated and estimated flow fields in the vertical cross-section.

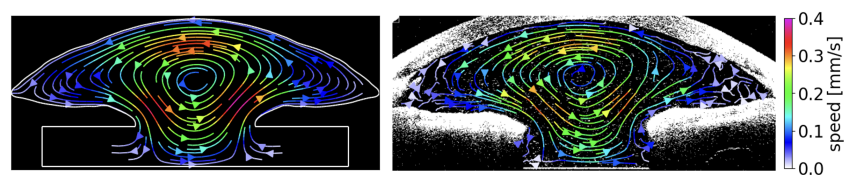


Figure 1: Comparison of flow fields in the vertical cross-section. (Left: Simulation, Right: Estimation)

The comparison demonstrates good agreement between the estimation and simulation.

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### References

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